GRAPH THEORY

Tutorial – 5

1) Prove or disprove: Every tree has at most one perfect matching.

2) Prove that every maximal matching in a graph G has at least $\alpha'(G)/2$ edges.

3) Let G be an X, Y-bigraph such that $|N(S)| > |S|$ whenever $\Phi \neq S \subset X$. Prove that every edge of G belongs to some matching that saturates X.

4) Prove that every bipartite graph G has a matching of size at least $e(G)/\Delta(G)$.

Use this to conclude that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least k.

5) Let G be a nontrivial simple graph. Prove that $\alpha(G) \leq n(G) - e(G)/\Delta(G)$.

Conclude that $\alpha(G) \leq n(G)/2$ when G also is regular.

6) For $k \geq 2$, prove that Q_k has at least $2^{2^{k-2}}$ perfect matching.

7) Let **A** = $(A_1, \ldots, A_m)$ be a collection of subsets of a set Y. A *system of distinct representatives* (SDR) for **A** is a set of distinct elements $a_1, \ldots, a_m$ in Y such that $a_i \in A_i$. Prove that **A** has an SDR if and only if $|\cup_{i \in S} A_i| \geq |S|$ for every $S \subseteq \{1,2,\ldots,m\}$.